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Steady-State Heat Transfer Analysis of Arbitrarily Coiled Buried Pipes

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The heat transfer from buried pipes of arbitrary shape is investigated. An approximate analytical technique, referred to here as the line source approximation (LSA), is developed and is easily applicable to pipes with complicated geometry. The essence of the method is to replace the pipe with a line source. The accuracy of the technique is verified by testing it against the known analytical result for a straight horizontal pipe. A spirally coiled pipe, sometimes proposed in ground-source heat pump designs, is used to demonstrate the applicability of this technique. The LSA is capable of providing detailed information, such as thermal interference, of the complicated geometry on the heat transfer process. The applicability of this technique is limited to steady-state heat conduction in the soil around the pipe.

Nomenclature

d	= coil diameter of the spiral arrangement
d_0	= pipe diameter
$\tilde{d}_0$	= nondimensional pipe diameter, d_0/h
g_0	= constant heat generation term, W
h	= (mean) burial depth
$\tilde{h}$	= nondimensional burial depth, h/d
k	= thermal conductivity of the soil
n	= number of spirally coiled loops
PF_{SS}	= steady-state performance factor
p_x, p_y	= coil pitches in the x and y directions
$\tilde{p}_x$, $\tilde{p}_y$	= nondimensional coil pitches in the x and y directions, p_x/d, p_y/d
q'	= strength of the line heat source (approximates the heat transfer per unit length of pipe)
q'_a	= analytic result for the heat transfer per unit length of pipe
r	= coil radius, $d/2$
r_0	= pipe radius, $d_0/2$
$\tilde{r}_0$	= nondimensional pipe radius, $\tilde{d}_0/2$
T_a	= ambient temperature
T_S	= ground surface temperature
T_1	= pipe wall temperature
$T(x, y, z)$	= soil temperature at a given location
x, y, z	= spatial coordinates
$\tilde{x}, \tilde{y}, \tilde{z}$	= nondimensional spatial coordinates, x/d, y/d, z/d
x_0, y_0, z_0	= coordinates of a variable point on the heat source
x', y', z'	= coordinates of a particular point on the spiral heat source, obtained by setting $\omega = \omega'$
ρ, θ	= polar coordinates
$\tilde{\rho}$	= nondimensional radial coordinate
ω	= parameter to generate the three-dimensional spiral shape
ω'	= particular value of ω denoting a particular location on the spiral

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I. Introduction

HEAT transfer from buried pipes, cables, and heat sources has been studied extensively in the past. The motivation for these studies has been wide ranging: energy dissipation from underground power cables and distribution transformers, studies of underground nuclear waste repositories that generate heat, underground rejection of waste heat from power plants, etc. Of late, a major motivation has been the design of efficient underground heat exchangers for use in ground-source heat pump systems.

The earliest analytical investigations in the area of buried pipes are by Ingersoll and Plass[1] and Eckert and Drake.[2] They use line source (or cylindrical source) theory to solve for the temperature profile around a buried horizontal straight pipe where both the pipe wall and the ground surface are isothermal. The major drawback of this approach is that one has to guess or assume the strength of the line source, which makes this approach dependent on empirical data. The steady-state heat conduction problem for a straight pipe with an isothermal ground surface has also been solved by Yovanovich[3] for the case of constant temperature at the pipe surface and by Thiyagarajan and Yovanovich[4] for the constant flux condition at the pipe surface. They make use of bicylindrical coordinates, which permits the boundary conditions to be expressed elegantly. They show that a straight pipe buried at a depth of three or more pipe radii can be treated as one having a constant temperature at the wall. Bicylindrical coordinates are also used by Bau and Sadhal[5] to study the coupled heat transfer processes inside the pipe and in the soil; and by Schneider[6] to study the heat transfer from a buried straight pipe to the ground convecting into an atmosphere at the ambient temperature.

Moisture migration from heated underground pipes can be an important factor in heat transfer in soils because it can significantly enhance the heat transfer based on a purely diffusive mechanism. Philip and de Vries[7] postulated a moisture transfer model consisting of a series of evaporation and condensation steps together with a discontinuous flow of liquid film. Several studies dealing with coupled heat and moisture flow in soils based on the theory of porous-media have also been made, the notable ones being Hartley and Black,[8] Baladi et al.,[9] and Radhakrishna et al.[10] Most of these also take into consideration transient effects and nonlinearities in soil properties, etc., and the governing equations are solved using numerical techniques. Mei[11] investigates the heat transfer from a buried pipe from the point of view of ground-source heat pump operation. He presents an original treatment of far-field boundary conditions. A finite difference scheme is used to solve coupled energy equations inside the pipe, in the pipe wall, and in the soil region.

All of the previously mentioned research has been performed essentially for a straight horizontal pipe. The trend has been toward including more physics into the problem to fully understand the complicated processes by which heat is transferred in soils. This previous work, however, cannot be directly applied to pipes that are coiled in arbitrary fashion (see Figs. 1 and 2). Such coiled pipe designs have recently been proposed for use as the underground heat exchanger in ground-source heat pumps. For coiled pipes, geometry can exert a significant influence on the heat transfer from the pipe.

The main focus of this research is to develop a simple analytic tool that can account for the influence of geometry on the heat transfer from the pipe. An approximate analytical technique, referred to here as the line source approximation (LSA), is developed to solve this problem. In LSA a coiled pipe is approximated by a line source that is coiled in exactly the same fashion as the centerline of the original pipe. The strength of the line source is computed by matching the pipe wall temperature to the average temperature around the line source at a radial location equal to the pipe radius. Then the strength of the line source (in watts per meter) is postulated to be a good approximation of the heat transfer per unit length of the pipe. The arbitrariness that is associated in specifying the strength of the line source in previous research employing line source theory[1,2] is eliminated, because the strength of the heat source is solved for directly and gives the required result for the heat transfer from the pipe. The accuracy of the LSA method is validated by comparing it to known analytical results for a straight pipe available in the literature. Since the primary objective of this research is to determine the influence of geometry on the heat transfer from the pipe, the physics of the problem are kept simple by considering a steady-state heat conduction problem in the soil around the pipe assuming constant thermal properties.

From the results of the LSA method applied to the spirally coiled pipe of Fig. 1, it is evident that the LSA method is capable of providing detailed information of the thermal interference over a single loop, as well as the heat transfer averaged over all of the loops as a function of the various geometric parameters. It should be emphasized that this particular geometry is chosen only as an example and the LSA method can be applied to pipes of circular cross sections coiled in any arbitrary fashion.

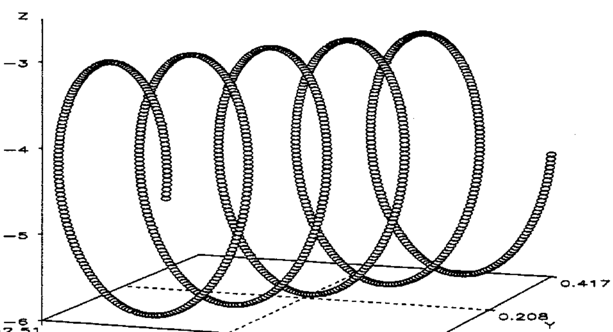

Fig. 1 Geometry of the spirally coiled pipe.

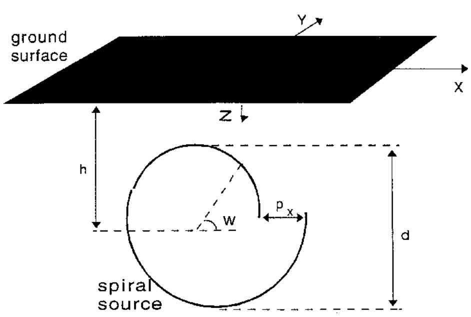

Fig. 2 Details of the spirally coiled geometry.

II. LSA Method Applied to an Infinitely Long Straight Pipe

In this section the details of the LSA method will be presented by applying it to a straight horizontal pipe buried in a semi-infinite medium. This pipe will be approximated by a straight line source. The pipe is assumed to have isothermal walls and the line source is assumed to have constant (but unknown) strength. The basic idea in the LSA method is to calculate the strength of the line source such that the average temperature around the line source at a radius equal to the pipe radius exactly matches the pipe wall temperature. Then the strength of the line source will be a good approximation of the heat transfer per unit length of the pipe.

In this section the solution for a straight line source is obtained by integration of the point source result. Then the details of the LSA method are presented by applying it to the line source. Finally, the accuracy of the LSA method is demonstrated by comparing the results with the analytical result for a straight pipe.

A. Exact Solution for a Straight Line Heat Source

This solution is sought in a semi-infinite medium corresponding to the typical soil domain. The domain is infinite in the x and y directions and semi-infinite in the z direction (the vertical direction) with $z = 0$ corresponding to the ground surface. The starting point of this derivation is the relation for the temperature distribution around a point source, of strength g_0, in a semi-infinite medium, given later. (Details of the derivation leading to this result are available in Carslaw and Jaeger[12] and Mukerji[13].)

$$T(x, y, z) = T_S + (g_0/4\pi k)[(1/r_1) - (1/r_2)] \tag{1}$$

where r_1 and r_2 are defined as

$$r_1 \equiv r_1(x, y, z) = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} \tag{2}$$

$$r_2 \equiv r_2(x, y, z) = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z + z_0)^2} \tag{3}$$

In the context of the line heat source, for which the solution is desired, (x_0, y_0, z_0) are the coordinates of any point on the line defining the heat source. Here it is assumed that the line source is of infinite length in the x direction and is buried at a depth h below the ground surface. Hence, $x_0 \in (-\infty, \infty)$, $z_0 = h$ and y_0 can be set to zero without any loss of generality. For an infinitesimally small element of the line source dx_0, the elemental strength dg_0 is given by $dg_0 = q'\, dx_0$, where q' is the strength of the line source in watts per meter. The temperature distribution around the line source can be obtained if g_0 in Eq. (1) is replaced by the previous expression for dg_0 and an integration is carried out over the infinite extent of the line source. Equation (4) gives the temperature distribution around an infinitely long straight line source:

$$T(x, y, z) - T_S = (q'/4\pi k)(I_1 - I_2) \tag{4}$$

where I_1 and I_2 are integrals defined as

$$I_1 \equiv \int_{-\infty}^{\infty} \frac{dx_0}{r_1} = \int_{-\infty}^{\infty} \frac{dx_0}{\sqrt{(x - x_0)^2 + y^2 + (z - h)^2}} \tag{5}$$

$$I_2 \equiv \int_{-\infty}^{\infty} \frac{dx_0}{r_2} = \int_{-\infty}^{\infty} \frac{dx_0}{\sqrt{(x - x_0)^2 + y^2 + (z + h)^2}} \tag{6}$$

The integrals I_1 and I_2 can be simplified by recognizing that the line source is infinitely long in the x direction, producing a temperature distribution that is independent of x. For simplicity, x can be prescribed to be zero without any loss of generality. Also, since the integrands are symmetric about $x_0 = 0$, the limits of integration can be defined over $(0, \infty)$ and the integral multiplied by 2. With a cylindrical geometry around the line source it is more convenient to locate the origin of the axes on the line source, instead of the ground surface, and express the result in polar coordinates. Accordingly, a new translated z axis, $\hat{z}$, can be defined, such that $\hat{z} = z - h$. The x and y axes remain unchanged. The y and $\hat{z}$ coordinates can be converted to their polar representation in $\boldsymbol{\rho}$ and θ coordinates as $y = \boldsymbol{\rho}\cos(\theta)$ and $\hat{z} = \boldsymbol{\rho}\sin(\theta)$. With these coordinate modifications, the integrals I_1 and I_2 can be evaluated analytically to produce

$$I_1 - I_2 = f(\rho, \theta) = \ell n[1 + (4h/\rho)[(h/\rho) + \sin\theta]] \quad (7)$$

A nondimensional version of the previous expression can also be defined as

$$I_1 - I_2 = \tilde{f}(\tilde{\rho}, \theta) = \ell n[1 + (4/\tilde{\rho})[(1/\tilde{\rho}) + \sin\theta]] \quad (8)$$

Where $\tilde{\rho}$ in the second case is the radial coordinate ρ nondimensionalized with the burial depth h. The temperature distribution around an infinitely long line heat source can now be expressed as

$$T(\tilde{\rho}, \theta) - T_S = (q'/4\pi\kappa)\tilde{f}(\tilde{\rho}, \theta) \quad (9)$$

It should be noted that $\tilde{\rho}$ and θ are the polar coordinates centered on the heat source. As can be expected, because of the infinite extent of the line heat source, the temperature field around it is two dimensional.

B. LSA Method Applied to a Straight Pipe

Here the LSA method is applied to a buried pipe in the form of a circular cylinder of radius r_0 (diameter d_0) and infinite length buried at a depth h below the ground surface. The pipe wall is isothermal at T_1 and the ground surface is isothermal at T_S. The line heat source is of infinite length and has the same burial depth h as the pipe centerline. The solution for this has already been obtained in Eq. (9).

The first step in the LSA method is to match T_1 at a radial distance of r_0 from the line source. However, we can see from Eq. (9) that this is not possible because the temperature around the line source depends on θ in addition to $\tilde{\rho}$. The next best option is to match the temperature averaged over θ to the pipe wall temperature. For this purpose we define a θ-integrated temperature $T_{av}(\tilde{\rho})$ as follows:

$$T_{av}(\tilde{\rho}) = \int_0^{2\pi} T(\tilde{\rho}, \theta)\, d\theta \quad (10)$$

The expression for $T_{av}(\tilde{\rho})$ follows from Eqs. (9) and (10):

$$T_{av}(\tilde{\rho}) - T_S = \frac{q'}{4\pi\kappa}\int_0^{2\pi} \tilde{f}(\tilde{\rho}, \theta)\, d\theta = \frac{q'}{4\pi\kappa}\tilde{f}_{av}(\tilde{\rho}) \quad (11)$$

From the form of $\tilde{f}(\tilde{\rho}, \theta)$, given by Eq. (8), it can be seen that the integration of over θ to obtain $\tilde{f}_{av}(\tilde{\rho})$ has to be done numerically.

To match the pipe wall temperature at a radial distance of r_0 from the line source requires $T_{av}(\tilde{r}_0) = T_1$. Here, $\tilde{r}_0$ is the nondimensional form of r_0 (i.e., r_0/h). This is necessary because of the corresponding nondimensionalization of ρ. Substituting this in the expression for average temperature, given by Eq. (11), produces

$$T_1 - T_S = (q'/4\pi k)\tilde{f}_{av}(\tilde{r}_0) \quad (12)$$

Equation (12) can be rearranged by making q' the subject of the formula. This step produces Eq. (13). Although this step is mathematically straightforward, it involves a subtle reinterpretation of q' from a problem parameter to a dependent variable:

$$q' = \frac{4\pi k(T_1 - T_S)}{\tilde{f}_{av}(\tilde{r}_0)} \quad (13)$$

The result obtained in Eq. (13) is an expression for the strength of an infinitely long line source in watts per meter, which produces temperature T_1 at a radial distance of r_0, at a depth h below the ground surface which is isothermal at T_S. The q' obtained from Eq. (13) will be a good approximation of the heat transfer per unit length from a cylindrical pipe of r_0 with an isothermal surface T_1 and its axis at an h below ground surface, which is at a constant T_S. This statement, relating the strength of the line source to the heat transfer from the pipe, is the very essence of the LSA method applied to a buried pipe.

Although the validity of the LSA method will be demonstrated rigorously in the next section, it should be noted that the temperature profile in the soil around a line source is the limit of the temperature profile around a circular cylinder as $r_0 \to 0$. Therefore, the LSA method can be expected to produce results of higher accuracy when applied to cylindrical pipes of smaller radii.

C. Validation of the LSA Method

The result obtained in Eq. (13) is to be compared to the analytic result for a straight pipe, which is taken from Incropera and DeWitt,[14] and reproduced in Eq. (14):

$$q'_a = \frac{2\pi k(T_1 - T_S)}{\cosh^{-1}(h/r_0)} \quad (14)$$

The subscript a denotes the analytic result. Using the nondimensional variables defined previously, the analytic result can be put in the following nondimensional form:

$$q'_a = \frac{2\pi k(T_1 - T_S)}{\cosh^{-1}(1/\tilde{r}_0)} \quad (15)$$

To compare the result obtained by the LSA method, Eq. (13), and the exact analytic result, Eq. (15), a simple ratio can be computed:

$$\frac{q'}{q'_a} = \frac{\cosh^{-1}(1/\tilde{r}_0)}{\tilde{f}_{av}(\tilde{r}_0)} \quad (16)$$

It can be seen that the entire expression in Eq. (16) depends on a single nondimensional variable $\tilde{r}_0$, which is the ratio of the pipe radius to its burial depth. For the LSA method to have high accuracy, the ratio q'/q'_a should be very close to unity (or 100%).

Figure 3 shows a plot of the error, $1 - q'/q'_a$, vs $\tilde{r}_0$. It can be seen from this plot that the LSA method is highly accurate. The error increases with increasing $\tilde{r}_0$, which is to be expected for larger pipe diameters or smaller burial depths. The largest pipes that are used for the ground heat exchanger in commercial ground source heat pump systems have a nominal diameter of 2 in. The minimum possible burial depths are about 3 ft, giving a maximum $\tilde{r}_0$ value of 0.0278. Values of $\tilde{r}_0$ less than this figure are marked in Fig. 3 as the possible region of heat pump operation. In this region the line source approximation has errors that are well below 0.01%. On the basis of this high accuracy, it can be concluded that the LSA method is a pow-

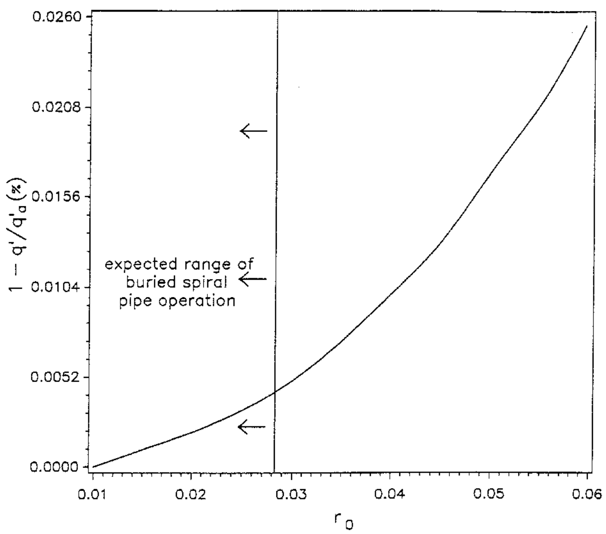

Fig. 3 Accuracy of the LSA method.

erful tool in estimating the steady-state heat transfer from buried pipes of circular cross sections.

III. LSA Method Applied to a Spirally Coiled Buried Pipe

In this section the capabilities of the LSA method will be demonstrated by applying it to the particular geometry of a spirally coiled pipe as shown in Fig. 1. The pipe has a diameter d_0 and forms a coil of advancing spiral loops. The coil is characterized by its mean burial depth h, coil diameter d, and the coil pitch p_x. Figure 2 shows these in detail for a single loop of the coil. As mentioned earlier, the spirally coiled pipe geometry shown in Figs. 1 and 2 is used only as an example to test the application of the LSA method. The LSA method is very versatile and can be applied to circular pipes coiled in any arbitrary fashion.

To apply the LSA method to the spirally coiled pipe, attention has to be focused on a line source coiled in exactly the same way as the centerline of the pipe. The temperature distribution around such a line source has to be first obtained. Following that, the details of the implementation of the LSA method are described.

A. Solution for a Spirally Coiled Line Source

To obtain the temperature distribution around a spirally coiled line source, the point source result given by Eqs. (1–3) has to be integrated over the geometry of the line source. Recalling that (x_0, y_0, z_0) is any point on the line heat source, the spirally coiled geometry of the source can be represented by parametric equations, in the parameter $\boldsymbol{\omega}$, of the form

$$x_0 = (d/2)[\cos(\omega) - 1] + (\omega p_x/2\pi) \tag{17}$$

$$y_0 = \omega p_y/2\pi \tag{18}$$

$$z_0 = (d/2)\sin(\omega) - h \tag{19}$$

The parameter ω can be viewed as an angular coordinate because as ω goes from 0 to 2π, one complete loop of the spiral is traversed.

To integrate the point source result, an infinitesimally small line element, of angular extent $d\omega$, is considered. Figure 4 presents a clearer picture of this line element. The elemental strength dg_0 of this line element can be approximated as $dg_0 = q's\, d\omega$, where q' is in watts/meter and s is the length along the spiral given by

$$s = \sqrt{\frac{p_x^2 + p_y^2}{4\pi^2} + \frac{d^2}{4} - \frac{dp_x}{2\pi}\sin(\omega)} \tag{20}$$

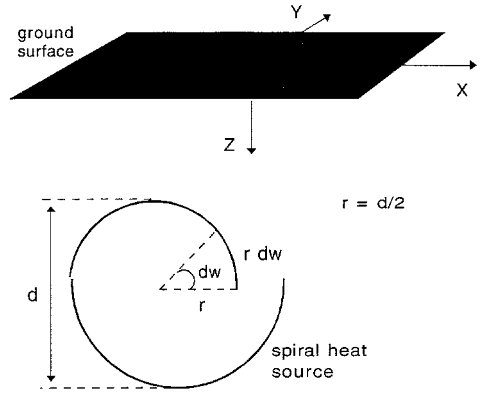

Fig. 4 Line element of the spiral heat source.

It should be noted from Eq. (20) that for small pitches ($p_x = p_y = 0$), $s = d/2 = r$; i.e., the elemental strength for a circular line source, $dg_0 = q'r\, d\omega$ is recovered.

Integration of Eq. (1) over ω produces the following expression for the temperature distribution around the spirally coiled line source:

$$T(x, y, z) - T_S = \frac{q'}{4\pi k}\left(\int_0^{n(2\pi)} \frac{s\, d\omega}{r_1} - \int_0^{n(2\pi)} \frac{s\, d\omega}{r_2}\right) \tag{21}$$

The parameter n in the upper limit of the integrals denotes the number of loops to be considered in the spiral arrangement. The functional forms of r_1 and r_2 can be obtained by substituting the parametric equations, Eqs. (17–19), into the definitions given in Eqs. (2) and (3):

$$r_1 = \sqrt{X_1^2 + Y_1^2 + Z_1^2} \quad \text{and} \quad r_2 = \sqrt{X_2^2 + Y_2^2 + Z_2^2} \tag{22}$$

where

$$X_1 = X_2 = x - (d/2)[\cos(\omega) - 1] + (\omega p_x/2\pi) \tag{23}$$

$$Y_1 = Y_2 = y - (\omega p_y/2\pi) \tag{24}$$

$$Z_1 = z - (d/2)\sin(\omega) + h \tag{25}$$

$$Z_2 = z + (d/2)\sin(\omega) - h \tag{26}$$

Equations (21–26) represent the temperature distribution around a spirally coiled line source of strength q' buried under a ground surface isothermal at T_S. The influence of the geometry on the temperature profile is conveyed by the functions r_1 and r_2.

B. Implementation of the LSA Method

Because of the somewhat complicated nature of the geometry, some notation needs to be clarified before proceeding with the derivation. As mentioned before (x_0, y_0, z_0) is a variable location on the spiral generated by the parameter ω. (x', y', z') refers to a particular point on the spiral generated by a given value of ω, denoted by ω'. (x, y, z) is any point around the spiral where the temperature is to be calculated.

Figure 5 clarifies the location of the points mentioned earlier. Since (x', y', z') is a point on the spiral, it can be obtained from the parametric equations, Eqs. (17–19), by setting $\omega = \omega'$:

$$x' = (d/2)[\cos(\omega') - 1] + (\omega' p_x/2\pi) \tag{27}$$

$$y' = \omega' p_y/2\pi \tag{28}$$

$$z' = (d/2)\sin(\omega') - h \tag{29}$$

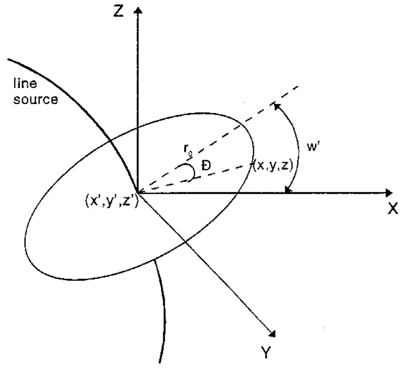

Fig. 5 Coordinate system around spiral line source.

Figure 5 also shows a circle of r_0 centered at the point (x', y', z') on the spiral. It should be noted that $\boldsymbol{\omega}'$, which determines the location of (x', y', z'), is also a measure of the angular rotation of the plane of the circle about the y axis, measured counterclockwise from the x–y plane. This circle is significant because it corresponds to the cross section of a spirally coiled pipe of r_0 that is to be approximated by the line source. The isothermal pipe wall T_1 is to be imposed on the points that lie on this circle. Once the points of interest are restricted to this circle, x, y, and z cease to be independent variables and become dependent on (x', y', z') and other geometric parameters. From Fig. 5 and simple trigonometric considerations

$$x - x' = r_0 \cos(\boldsymbol{\theta})\cos(\omega') \tag{30}$$

$$y - y' = r_0 \sin(\theta) \tag{31}$$

$$z - z' = r_0 \cos(\theta)\sin(\omega') \tag{32}$$

It can be seen from Eqs. (30–32) that x, y, and z depend on x', y', and z', which themselves depend on ω' via Eqs. (27–29). Hence, x, y, and z depend on ω' and geometric parameters r_0, d, h, p_x, and p_y. θ is averaged out in the same way as was done for the straight line source by integration over $\theta \in (0, 2\pi)$ and dividing the result by 2π. Once x, y, and z are restricted to radial locations at a distance of r_0 from the spiral line source where the temperature is T_1, Eq. (21) can be modified as

$$T_1 - T_S = \frac{q'}{4\pi k}\left[\int_0^{n(2\pi)} \frac{s\,\mathrm{d}\omega}{r_1} - \int_0^{n(2\pi)} \frac{s\,\mathrm{d}\omega}{r_2}\right] \tag{33}$$

The parameter q' in Eq. (33) is now reinterpreted as a dependent variable. Rearrangement of Eq. (33) with q' as the subject of the formula produces Eq. (34):

$$q'(\omega', d, h, r_0, p_x, p_y) = 4\pi k(T_1 - T_S)\Bigg/\left[\int_0^{n(2\pi)} \frac{s\,\mathrm{d}\omega}{r_1} - \int_0^{n(2\pi)} \frac{s\,\mathrm{d}\omega}{r_2}\right] \tag{34}$$

Heat transfer per unit length of the coiled pipe q' is thus a function of location on the spiral ω' and different geometric factors. Here, r_1 and r_2 are still given by Eq. (22), but with x, y, and z restricted, via Eqs. (30–32), to radial locations at a distance of r_0 from the spirally coiled line source.

The result obtained in Eq. (34) is nondimensionalized with respect to d. As a clarification, it must be mentioned that $r = d/2$ and $r_0 = d_0/2$. The nondimensional variables are defined as

$$\tilde{h} = \frac{h}{d}, \quad \tilde{d}_0 = \frac{d_0}{d}, \quad \tilde{p}_x = \frac{p_x}{d}, \quad \tilde{p}_y = \frac{p_y}{d} \tag{35}$$

$$\tilde{s} = \frac{s}{d} = \sqrt{\frac{\tilde{p}_x^2 + \tilde{p}_y^2}{4\pi^2} + \frac{1}{4} - \frac{\tilde{p}_x}{2\pi}\sin(\omega)} \tag{36}$$

In practical applications p_y is not a design variable and is fixed based on d_0; it is specified as $p_y = d_0$, i.e., the loops stack up in the y direction, advancing a distance equal to the pipe diameter between successive loops. p_y can thus be excluded from the parameter list in the functional form of q' and treated as a mere number.

Making use of the nondimensional variables defined earlier, the expression for q' becomes

$$q'(\omega', \tilde{h}, \tilde{d}_0, \tilde{p}_x) = 8\pi k(T_1 - T_S)\Bigg/\left[\int_0^{n(2\pi)} \frac{\tilde{s}\,\mathrm{d}\omega}{\tilde{r}_1} - \int_0^{n(2\pi)} \frac{\tilde{s}\,\mathrm{d}\omega}{\tilde{r}_2}\right] \tag{37}$$

Here, r_1 and r_2 are defined as in Eq. (22), but in terms of nondimensional variables as

$$\tilde{r}_1 = \sqrt{\tilde{X}_1^2 + \tilde{Y}_1^2 + \tilde{Z}_1^2} \quad \text{and} \quad \tilde{r}_2 = \sqrt{\tilde{X}_2^2 + \tilde{Y}_2^2 + \tilde{Z}_2^2} \tag{38}$$

where

$$\tilde{X}_1 = \tilde{X}_2 = \tilde{x} - \tfrac{1}{2}[\cos(\omega) - 1] + (\omega\tilde{p}_x/2\pi) \tag{39}$$

$$\tilde{Y}_1 = \tilde{Y}_2 = \tilde{y} - (\omega\tilde{p}_y/2\pi) \tag{40}$$

$$\tilde{Z}_1 = \tilde{z} - \tfrac{1}{2}\sin(\omega) + \tilde{h} \tag{41}$$

$$\tilde{Z}_2 = \tilde{z} + \tfrac{1}{2}\sin(\omega) - \tilde{h} \tag{42}$$

The variables $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$ that appear in Eqs. (39–42) denote particular locations around the spiral line source. They are specified in terms of ω' by Eqs. (43–45), which are the nondimensional analogues of Eqs. (30–32).

$$\tilde{x} = (\tilde{d}_0/2)\cos(\theta)\cos(\omega') + x' \tag{43}$$

$$\tilde{y} = (\tilde{d}_0/2)\sin(\theta) + y' \tag{44}$$

$$\tilde{z} = (\tilde{d}_0/2)\cos(\theta)\sin(\omega') + z' \tag{45}$$

Here, x', y', and z' are locations on the line source obtained by setting $\omega = \omega'$ in the parametric equations. These are given by Eqs. (46–48), which are the nondimensional version of Eqs. (27–29):

$$x' = \tfrac{1}{2}[\cos(\omega') - 1] + (\omega'\tilde{p}_x/2\pi) \tag{46}$$

$$y' = \omega'\tilde{p}_y/2\pi \tag{47}$$

$$z' = \tfrac{1}{2}\sin(\omega') - \tilde{h} \tag{48}$$

Equations (37–48) are the complete solution for the heat transfer per unit length from a spirally coiled pipe obtained by the LSA method. As would be expected from the complicated nature of the geometry, the equations, too, seem rather complex; but the complexity is in the details and not in the mathematics required for a solution.

IV. Results and Discussions for Spirally Coiled Buried Pipes

The steady-state heat transfer results for a coiled pipe are presented in terms of PF_{SS}. PF_{SS} is defined as the ratio of the steady-state heat transfer per unit length of the spirally coiled pipe to the steady-state heat transfer per unit length of straight pipe having the same diameter and located at the same burial depth. The steady-state analytical result is given by Eq. (14) and can be written in nondimensional form as

$$q'_a = \frac{2\pi k(T_1 - T_S)}{\cosh^{-1}(2\tilde{h}/\tilde{d}_0)} \tag{49}$$

The expression for PF_{SS} now becomes

$$PF_{SS} = \frac{q'}{q'_a} = 4\cosh^{-1}(2\tilde{h}/\tilde{d}_0) \Bigg/ \left[\int_0^{n(2\pi)} \frac{\tilde{s}\,d\boldsymbol{\omega}}{\tilde{r}_1} - \int_0^{n(2\pi)} \frac{\tilde{s}\,d\omega}{\tilde{r}_2} \right] \tag{50}$$

where $\tilde{r}_1$ and $\tilde{r}_2$ are given by Eqs. (38–48). As can be seen from Eq. (50), PF_{SS} is a function of only a location coordinate ω' and three nondimensional geometric parameters, $\tilde{h}$, $\tilde{d}_0$, and $\tilde{p}_x$. Putting the results in the form of PF_{SS} isolates the influence of geometry on heat transfer, which has been the main focus of this research. An implicit assumption is made that the influence of nongeometric factors on heat transfer, for example, soil texture, moisture content, variation in thermal properties, etc., is roughly the same order of magnitude for both straight and spirally coiled pipes.

Figure 6a shows the location of the middle loop, a solid line, in a coil of 81 loops with only the six neighboring loops on either side shown as dashed lines. Figure 6b shows the variation of PF_{SS} over the middle loop as ω' goes from 0 to 2π. The rest of the geometric factors are held constant at the base design values: $h = 4.5$ ft, $d_0 = 1$ in., $p_x = 1$ ft, and $d = 3$ ft, yielding nondimensional parameters of $\tilde{h} = 1.5$, $\tilde{d}_0 = 0.0278$, and $\tilde{p}_x = 0.333$. It can be seen that points on the pipe closer to the ground ($\omega' = 0.5\pi$) give off more heat than points deeper underground ($\omega' = 1.5\pi$), because the thermal resistance increases as the distance from the ground surface. The effect of thermal interference is quite clear, as points where the pipes are close together have much lower values of PF_{SS} and vice versa. It can be seen from Fig. 6a that there are four points centered around $\omega' = 0.5\pi$ and six points centered around $\omega' = 1.5\pi$, where two coils actually cross each other. The performance curve of Fig. 6b shows sharp drops at exactly these points of nearest crossing, which is to be expected because of the severe thermal interference at these points.

Figure 7 shows the effect of the geometric parameters, $\tilde{h}$, $\tilde{d}_0$, and $\tilde{p}_x$ on the performance averaged over all 81 loops. The decreasing trend indicated by Fig. 7a shows that the performance of a spiral pipe drops off faster than that of a straight pipe with increasing burial depth. From Fig. 7b it appears that the gain in performance, to be expected by using a pipe of larger diameter, is offset by the increased thermal interference from the adjacent loops and hence the slight decrease in performance with increasing pipe diameters. Figure 7c suggests that the pitch parameter $\tilde{p}_x$ is the most significant influence on the spiral pipe performance. A larger pitch leads to an opening out of the spiral pipe, resulting in reduced thermal interference, and hence, higher performance.

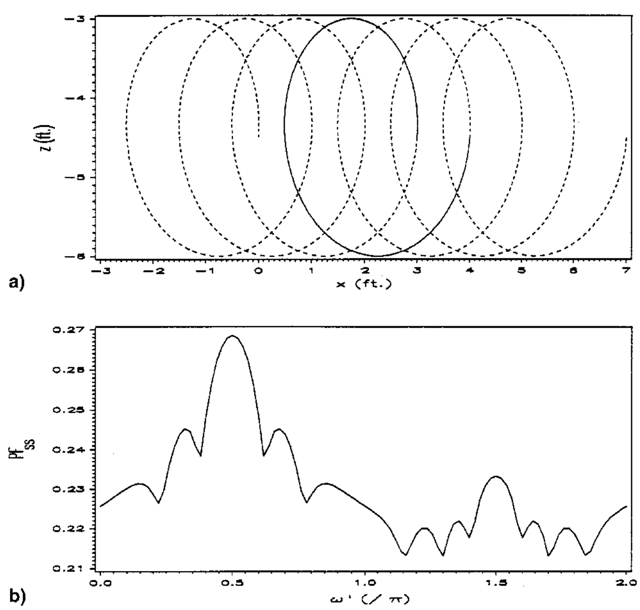

Fig. 6 Heat transfer variation over a loop: a) location of the loop and b) performance curve.

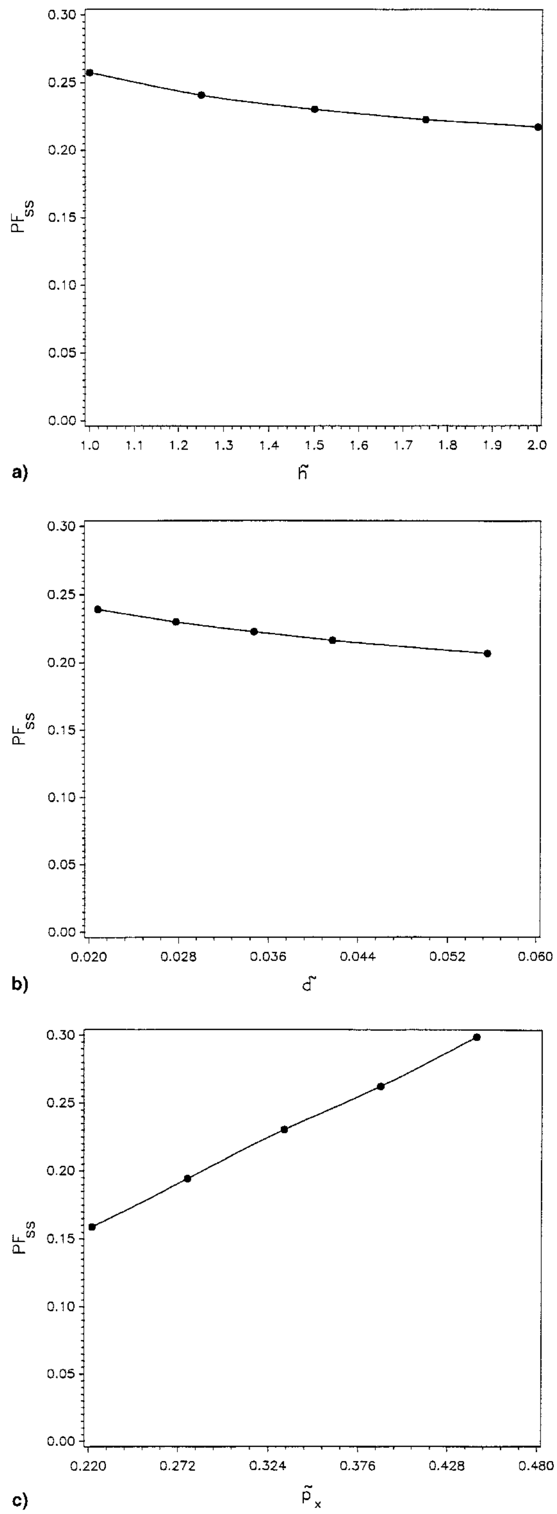

Fig. 7 Heat transfer variation with geometric parameters: a) burial depth, b) pipe diameter, and c) coil pitch.

V. Concluding Remarks

An approximate analytical method referred to here as LSA is developed and can be used to calculate the heat transfer from buried pipes of circular cross sections. The LSA method is found to be highly accurate when it is applied to a straight pipe and the results compare well with the analytic results for a straight pipe available in the literature.

The LSA method is then used to calculate the heat transfer from a spirally coiled pipe. These results show the effect of thermal interference in great detail. The ratio of coil pitch to coil diameter is found to be the dominant geometric influence on the heat transfer performance. For the typical base design values used in the calculations, the heat transfer per unit length of a spirally coiled pipe was only about 23% of that of a straight pipe.

It should be noted that the spirally coiled geometry described here is chosen only as an example. The LSA method is quite general and can be applied to buried pipes of circular cross sections coiled in any arbitrary fashion.

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